Analysis of Flexible Hysteroscope Rigidity and Ovalization

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1 Problem

A flexible hysteroscope sheath should be able to withstand the bending required by a physician within the uterus without buckling or transferring excessive stress upon the internal channels and instruments. A general setup is sketch in Figure 1. In order to ensure that this is the case, we aim to find a range for the Young's Modulus and the minimum yield stress of a flexible hysteroscope sheath.

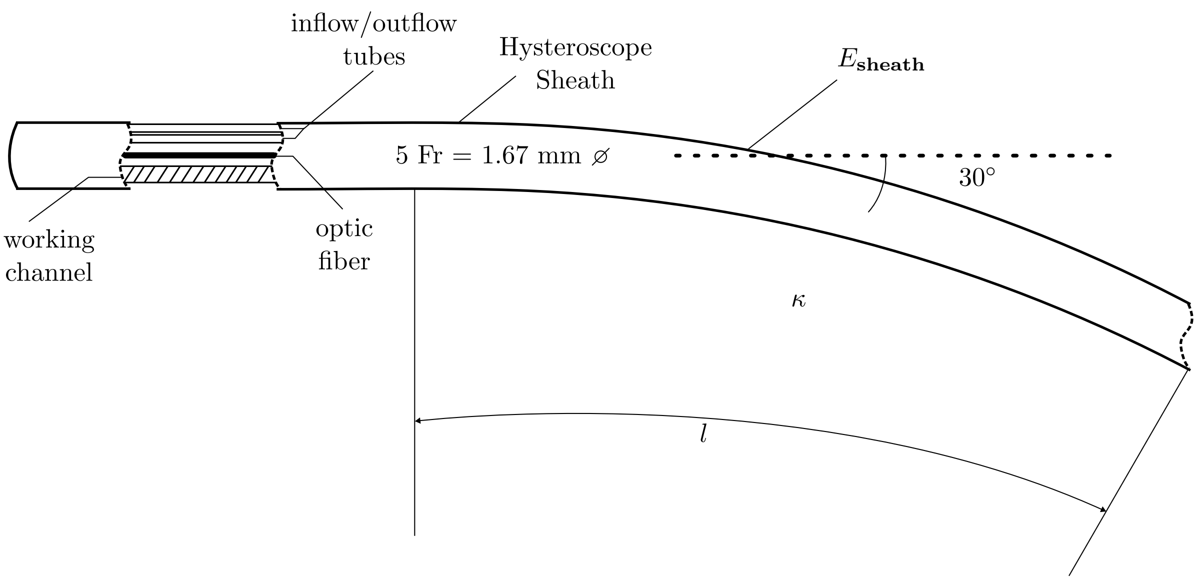

Figure 1: Sketch of bent hysteroscope sheath containing essential elements.

2 Data and Diagrams

Parameters	Value	Description
r_o	0.6 mm	Outer radius of inner fluid tube. Shown in Figures 3, 4
r_i	0.4 mm	Inner radius of inner fluid tube. Shown in Figures 3, 4
T_i	$r_o - r_i = 0.2$ mm	Thickness of inner fluid tube. Shown in Figures 3, 4
R	0.835 mm	Radius of hysteroscope sheath (Assumed 5 Fr). Shown in Figures 2, 3, 4
T	0.1 mm	Thickness of hysteroscope sheath. Shown in Figures 3, 4
l	5 cm	Length of curved section of hysteroscope sheath/inner tube [5]. Shown in Figure 2
ν_s	0.25	Assumed Poisson's Ratio of the sheath
ν_i	0.49	Assumed Poisson's Ratio of inner tube (rubber-like)
E_i	600 MPa	Young's Modulus of inner tube [1]
$\sigma_{\text{yield},i}$	20 MPa	Yield stress of inner tube [1]
$\sigma_{\text{yield},s}$	?	Yield stress of sheath
ζ_c	0.05	Critical flattening ratio

Table 1: The values for the inner fluid tube were chosen because it tends to have the highest Young's modulus and lowest yield stress according to [1]. The other potentially weakest component might be the optic fiber, but this was determined to have a significantly higher yield stress based on calculations from diameter given in [4] and elasticity modulus given in [2].

Variables	Value	Description
E_s	?	Young's Modulus of sheath
ζ	?	Flattening ratio of sheath cross section
ε_s	?	Hoop strain of sheath
ε_z	?	Axial strain of sheath
σ_s	?	Hoop stress of sheath
σ_z	?	Axial stress of sheath
$\tau_{\max}$	?	Maximum shear stress between inner tube and sheath

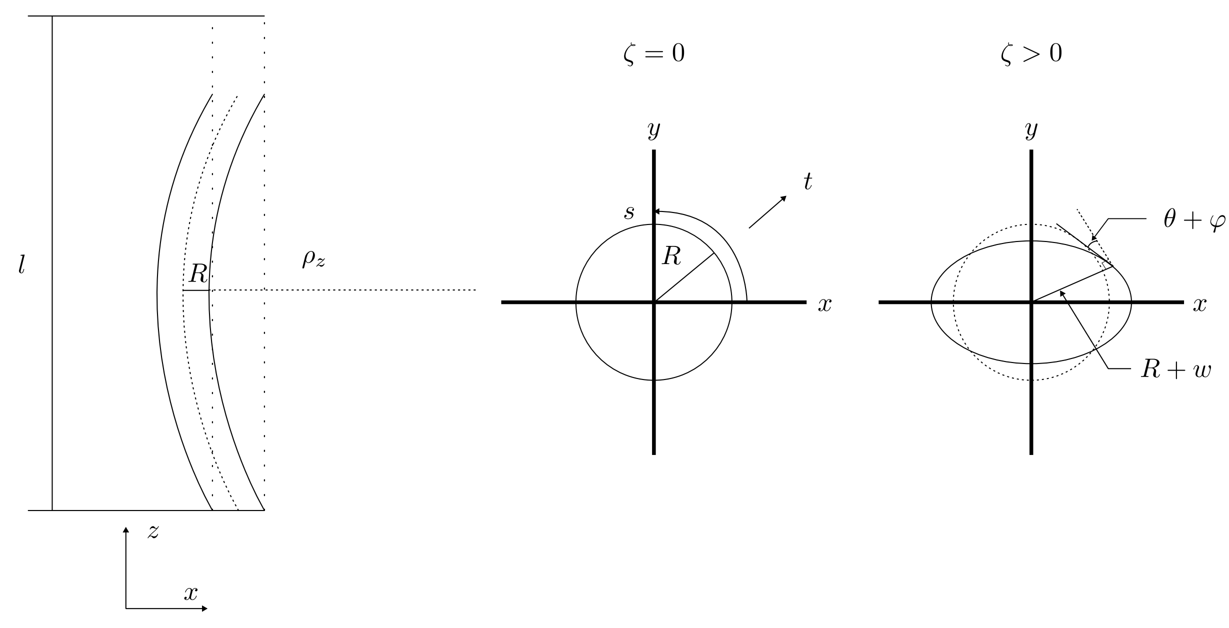

Figure 2: The tube ovalization is pictured in three ways. On the left, the curvature of the tube is shown with a radius of ρ_z or a curvature of $\kappa = \frac{1}{\rho_z}$. On the left, a dotted line also shows the straightened out tube to indicate that its full length is l. The two cross sections on the right show the ovalization of the tube under bending. We define the s- and t-axes to be the arc distance and radial position away from the z-axis respectively, as shown in the center case where $\zeta = 0$. The flattening ratio, ζ, shouldn't cross a critical threshold in order to ensure that buckling doesn't occur. In the case where $\zeta > 0$, we define $w(s)$ to be the change in the distance from the axial center to the edge of the tube at a given s along the tube. We define $\varphi(s)$ to be the change in the angle of the tangent plane from the undeformed circle.

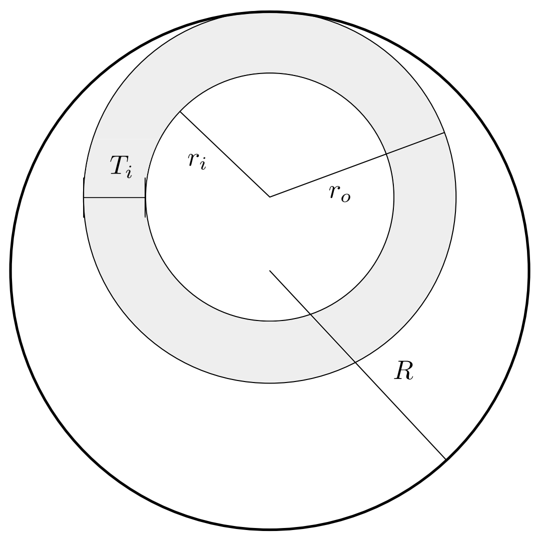

Figure 3: We assume that the inner tube will be pressed against the sheath and there will be a contact force that will exert a shear stress on both elements.

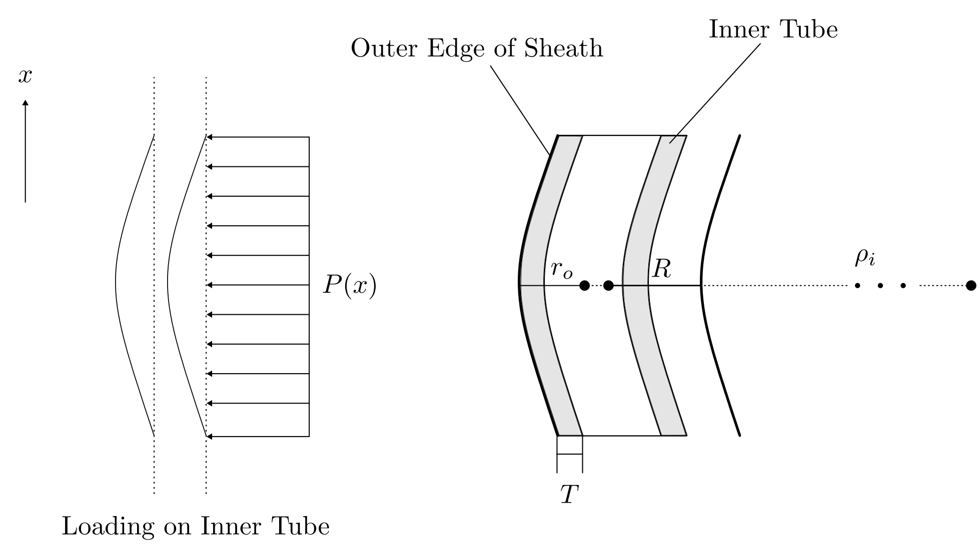

Figure 4: We assume a fixed curvature κ for the whole sheath and tube. We define $P(x)$ to be the force profile on the inner tube in order to achieve this curvature, as shown on the left. Note that $P(x)$ is not necessarily constant. The sketch on the right shows the radius of the inner tube's curvature, ρ_i.

3 Assumptions

In order to simplify analysis, we will assume that the sheath is thin walled and we can model the contact between the inner tube and sheath as a line load. In order to simplify the analysis of ovalization, we will also assume that ζ is close to zero. We also assume the force profile on the inner tube is quadratic.

4 Theory

4.1 Buckling Analysis of Sheath

We use the energy minimization method proposed by [7] to model the ovalization of our hysteroscope sheath. In this approach, we will minimize the strain energy, U, with respect to ζ in order to determine flattening. The strain energy is given by,

$$U = 2\int_0^{2\pi}\int_{-T/2}^{T/2}(\omega_1+\omega_2)\mathrm{d}t\mathrm{d}s, \tag{1}$$

where ω_1 and ω_2 are given by,

$$\omega_1 = \frac{2}{9}E_s\int_0^{\varepsilon_z}(2\varepsilon_z-\varepsilon_s)\mathrm{d}\varepsilon_z + \int_0^{\varepsilon_z}(2\varepsilon_s-\varepsilon_z)\mathrm{d}\varepsilon_s \tag{2}$$

$$\omega_2 = \frac{E_s}{6(1-2\nu_s)}(\varepsilon_z+\varepsilon_s)^2 + 2\varepsilon_s\varepsilon_z. \tag{3}$$

Axial stress is given by,

$$\varepsilon_z = \mathring{\varepsilon}_z + \frac{t}{\rho_z}\sin(\theta+\varphi), \tag{4}$$

where ρ_z is the radius of the curvature of the sheath. We expect for our hysteroscope to be able to turn 30° [6], so we set $\rho_z = l/(\frac{\pi}{6})$. As shown in Figure 2, φ is the angle of deflection from the original circular geometry due to ovalization. Here t represents the radial axis, as shown in Figure 2. In [7], $\mathring{\varepsilon}_z$ is defined as,

$$\mathring{\varepsilon}_z = \frac{x}{\rho_z} - 1 = \frac{R\cos(\theta+\varphi)}{\rho_z} - 1. \tag{5}$$

Similarly for hoop strain, we have,

$$\mathring{\varepsilon}_s = 2\pi R\left(\frac{1}{R}+\frac{\mathrm{d}\varphi}{\mathrm{d}s}\right) - 1 \tag{6}$$

$$\varepsilon_s = \frac{\mathring{\varepsilon}_s + t\frac{\mathrm{d}\varphi}{\mathrm{d}s}}{1+\frac{t}{R}}. \tag{7}$$

For a circular tube, [7] gives us,

$$\frac{\mathrm{d}\varphi}{\mathrm{d}s} = \frac{3\zeta}{R}\cos\left(\frac{2s}{R}\right). \tag{8}$$

Finally, in order to find ζ for a given geometry and material properties as laid out above, we must minimize U by solving,

$$\frac{\partial U}{\partial \zeta} = 0. \tag{9}$$

We can use our critical flattening ratio, ζ_c, to then compute the minimum Young's modulus for our sheath, E_s.

Once we have this, we can back calculate the shear and hoop stress with

$$\sigma_z = \frac{2}{9}\frac{E}{\bar{\varepsilon}}(2\varepsilon_z - \varepsilon_s) + \frac{E}{3(1-2\nu_s)}(\varepsilon_z + \varepsilon_s) \tag{10}$$

$$\sigma_s = \frac{2}{9}\frac{E}{\bar{\varepsilon}}(2\varepsilon_s - \varepsilon_z) + \frac{E}{3(1-2\nu_s)}(\varepsilon_z + \varepsilon_s) \tag{11}$$

$$\tag{12}$$

where the effective strain, $\bar{\varepsilon}$, is given by,

$$\bar{\varepsilon} = \frac{2}{3}\sqrt{\varepsilon_z^2 + \varepsilon_s^2 - \varepsilon_z\varepsilon_s}. \tag{13}$$

The yield stress of our chosen hysteroscope material should not be below either σ_z or σ_s.

4.2 Contact between Sheath and Inner Tube

In order to determine how much shear stress will be transferred to the inner tube, we can use a Hertz line contact, as laid out by [3]. First, we will need to find a force profile, as shown in Figure 4. We will assume a quadratic force profile.

$$P(x) = ax^2 \tag{14}$$

We know that the total moment acting on the pipe must be $M(l) = E_i I \kappa$ where the second moment of area, I, is given by

$$I = \frac{\pi}{2}(r_o^4 - r_i^4) \tag{15}$$

and the curvature, κ, is

$$\kappa = \frac{1}{\rho_i} = \frac{1}{2R - r_o + \rho_z}. \tag{16}$$

From here, we can find a to determine our force profile.

$$M(l) = \int_0^l P(x)x\,\mathrm{d}x \tag{17}$$

$$\iff E_i I \kappa = \int_0^l P(x)x\,\mathrm{d}x \tag{18}$$

$$\iff a = \frac{4E_i I \kappa}{l^4} \tag{19}$$

Now that we have a force profile, we can directly use the equations provided by [3] to find the half width of contact,

$$b = \sqrt{\frac{4PR_c}{\pi E_c}}. \tag{20}$$

Here, R_c and E_c are the relative radius and contact modulus respectively, given by,

$$\frac{1}{E_c} = \frac{1-\nu_s^2}{E_s} + \frac{1-\nu_i^2}{E_i} \tag{21}$$

$$\frac{1}{R_c} = \frac{1}{R} + \frac{1}{r_o}. \tag{22}$$

Finally, we can calculate the maximum pressure,

$$p = \frac{2P(l)}{\pi b} \tag{23}$$

and the maximum shear stress is given by $\tau_{\max} = 0.30p$. We can then ensure that our chosen E_s doesn't exceed the maximum required to ensure that $\sigma_{\text{yield},i} > \tau_{\max}$.

5 Solution

Both solutions were computed using the SymPy symbolic computation library in Python.

5.1 Buckling Analysis of Sheath

```python
from sympy import *

rho_z, R, T, nu, t, s, E, zeta = symbols("\\rho_z R T \\nu t s E \\zeta")
theta = s / R
phi = 3 * zeta / 2 * sin(2 * s / R)

eps_z = R * cos(theta + phi) / rho_z - 1 + t / rho_z * sin(theta + phi)

eps_s0 = 2 * pi * R * (1 / R + phi.diff(s)) - 1
eps_s = (eps_s0 + t * phi.diff(s)) / (1 + t / R)

omega_1 = 2 * E / 9 * (eps_z**2 - eps_z * eps_s) + 2 * eps_s * eps_z - eps_z**2 / 2
omega_2 = E / 6 * (1 - 2 * nu) * (eps_z + eps_s) ** 2 + 2 * eps_s * eps_z

U = 2 * integrate(integrate(omega_1 + omega_2, (t, -T / 2, T / 2)), (s, 0, 2 * pi))

l = 0.05
vals = {
    rho_z: 6 / pi.evalf() * l,
    R: 1.67e-3 / 2,
    T: 1e-4,
    nu: 0.49,
}

eq = Eq(U.subs(vals).diff(zeta).simplify(), 0)

# Assume zeta is really small and solve
zeta_sol = solve(eq, zeta)[0].subs({zeta: 0}).doit()

# We can now finally solve for a minimum E
E_min_expr = solve(zeta_sol - zeta, E)[0]

# Assume a maximum zeta of 0.05
E_min = E_min_expr.subs({zeta: 0.05}).evalf()

eps_eff = 2 / 3 * sqrt(eps_z**2 + eps_s**2 - eps_z * eps_s)

sigma_z = 2 * (2 * eps_z - eps_s) / 9 / eps_eff + E / (3 * (1 - nu**2)) * (
    eps_z + eps_s
)
sigma_s = 2 * (2 * eps_s - eps_z) / 9 / eps_eff + E / (3 * (1 - nu**2)) * (
    eps_z + eps_s
)

print("Minimum E:", E_min)

# The maximum stress will be in the inner and outer parts of the curve furthest
# from the center of the tube
print(
```

```
    "Max sigma_z:", sigma_z.subs({s: pi, t: R + T / 2, zeta: 0.05}).subs(vals).evalf()
)
print(
    "Max sigma_s:", sigma_s.subs({s: pi, t: R + T / 2, zeta: 0.05}).subs(vals).evalf()
)
```

5.2 Contact between Sheath and Inner Tube

```
from sympy import *

ro = 1.2e-3 / 2
ri = 0.8e-3 / 2
Ti = ro - ri
R = 1.67e-3 / 2
Rc = (1 / ro - 1 / R) ** -1
l = 5e-2
rho_z = 6 / pi.evalf() * l

nu_s = 0.25
nu_i = 0.49

Ei = 600e6
Es = symbols("E_s")

Ec = ((1 - nu_i**2) / Ei + (1 - nu_s**2) / Es) ** -1

I = pi / 2 * (ro**4 - ri**4)
rho_i = 2 * R - ro + rho_z
kappa = 1 / rho_i
P = 4 * Ei * I * kappa / l**2

b = ((4 * P * Rc) / (pi * Ec)) ** (1 / 2)
p = 2 * P / (pi * b)

print("Maximum Shear Stress:", 0.3 * p)
```

6 Results

6.1 Buckling Analysis of Sheath

The output from the code was:

```
Minimum E: 0.0814213097754493
Max sigma_z: 0.498050249871858*E - 0.497280150807378
Max sigma_s: 0.498050249871858*E + 0.633175733104086
```

Because the minimum E_s is so small, on the order of 10^-2 Pa, we can conclude that there is no practical lower bound for E_s. We do, however get an expression for the stress in terms of E_s, so we must ensure that for our chosen material,

$$\boxed{\sigma_{\text{yield},s} > 0.498050249871858 E_s + 0.633175733104086}. \tag{24}$$

6.2 Contact between Sheath and Inner Tube

From the code, we get

$$\boxed{\tau_{\max} = \frac{0.310181027837667}{\sqrt{5.58340369219741 \times 10^{-12} + \frac{0.00413299720602848}{E_s}}}} \tag{25}$$

However,

$$\lim_{E_s \to \infty} \tau_{\max} \approx 131,270 \text{ Pa} \ll \sigma_{\text{yield},i}, \tag{26}$$

so we also don't have to worry about an upper bound for E_s.

Thus, we have shown that E_s need not be restricted by an upper or lower bound, but the relationship in Equation 24 must be maintained in order to ensure that a hypothetical flexible sheath doesn't buckle. We do not need to worry about excessive stress on the inner tubing, which was found to be the weakest internal component, because it will not be loaded with shear stress exceeding its yield stress no matter our choice of E_s.

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Problem: Determine the maximum angle Θ that a hysteroscope sheath can be at with respect to the cervical canal without the distal end of the sheath leaving the vicinity of the fundus.

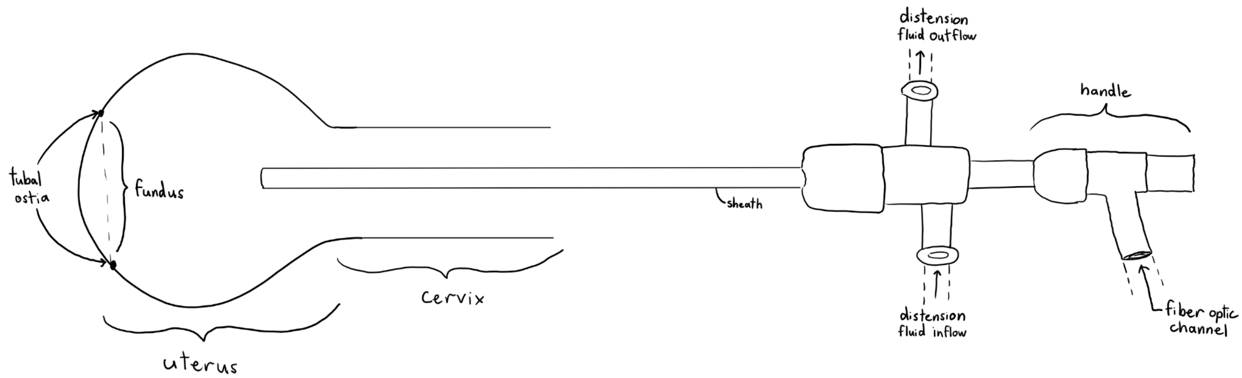

Figure 1. Diagram of a hysteroscope sheath inserted into the cervical canal.

Data and Diagram:

Variables	**Description**	**Value**
d_{io}	Intra-ostial distance (Bromer et al., 2007)	28.8 mm → 0.0288 m
r_u	Radius of the uterus (Refaey, n.d.)	2.5 cm → 0.025 m
l_c	Length of the cervical canal (Prendiville & Sankaranarayanan, 2017)	4 cm → 0.04 m
w_c	Width of the cervical canal (Prendiville & Sankaranarayanan, 2017)	3 cm → 0.03 m
l_{tot}	Distance from the center of the proximal end of the cervical canal to the center of the fundus	?
y	Vertical distance from the center of the cervical canal to the tubal ostia	?
Θ	Angle of the sheath with respect to the cervical	?

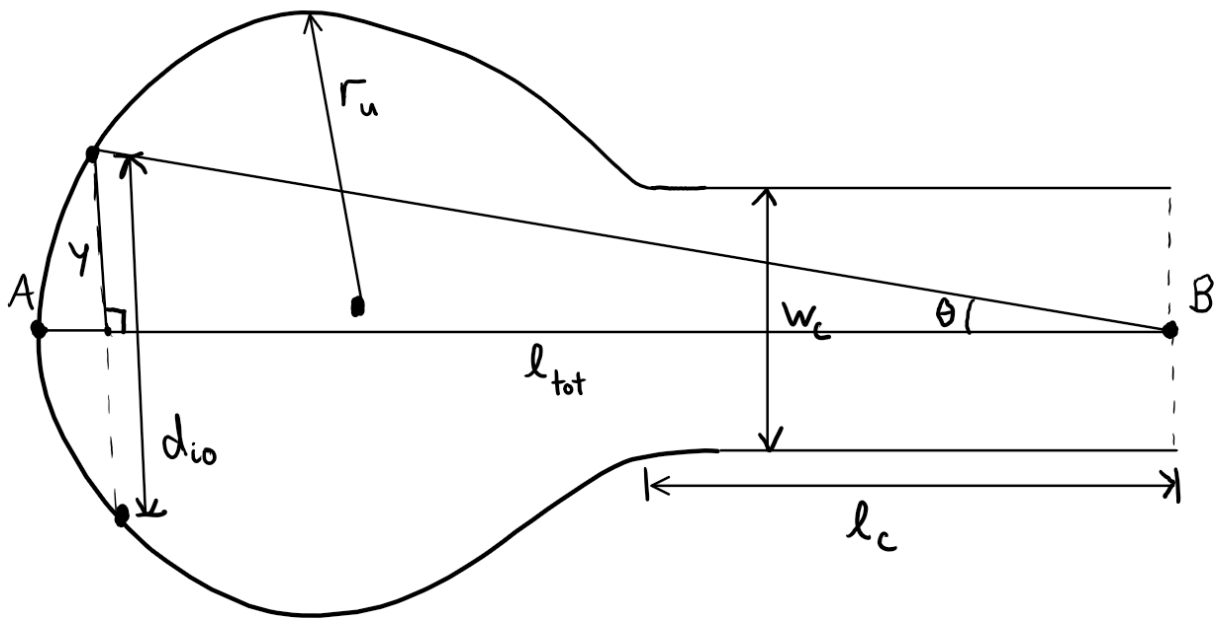

Figure 2. Diagram of the uterus and cervical canal illustrating the variables of the analysis.

Assumptions:

1. The uterus can be modeled as a sphere with a 2.5 cm radius.
2. The cervical canal can be modeled as a cylinder with a 3 cm diameter and a 4 cm length.
3. The uterus and cervical canal are symmetrical about the line segment AB.

Theory: tangent of an angle in a right triangle

- Tangent of the angle of the sheath with respect to the cervical canal (I)

$$\tan\theta = \frac{y}{l_{tot}}$$

Solution:

1. Divide the intra-ostial distance d_{io} by 2 to determine the value of y (II).

$$y = \frac{d_{io}}{2}$$

2. Add the diameter of the uterus $2r_u$ to the length of the cervical canal l_c to determine the value of l_{tot} (III).

$$l_{tot} = l_c + 2r_u$$

3. Take the inverse tangent of both sides of equation I (IV).

$$\theta = \tan^{-1}\left(\frac{y}{\ell_{tot}}\right)$$

4. Plug equations II and III into equation IV (V).

$$\theta = \tan^{-1} \frac{d_{io}}{2(\ell_c + 2r_u)}$$

5. Solve equation V.

$$\theta = \tan^{-1}\left(\frac{0.0288\ m}{2(0.04\ m + 2(0.025\ m))}\right)$$

$$\theta = 9.090°$$

Results and Discussion: Under the assumptions, the maximum angle that the sheath can be at with respect to the cervical canal without the distal end of the sheath leaving the vicinity of the fundus is approximately 9 degrees. This angle would decrease as the radius of the uterus increases or the length of the cervical canal increases. The angle would increase as the intra-ostial distance increases.

Works Cited

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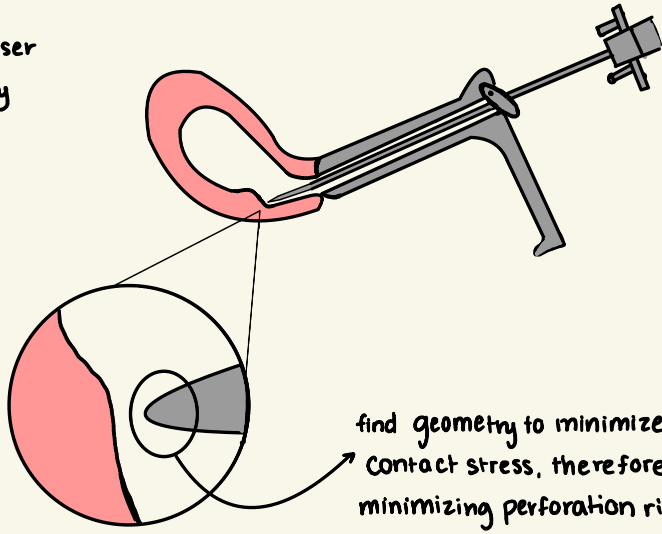
Problem: When performing a polypectomy, for a given insertion force, what tip geometry of the tissue removal device minimizes possible contact stress on the uterine **myometrium**

- Given insertion force F from user
- Design variable → tip geometry (shape + size parameters)



find geometry to minimize contact stress, therefore minimizing perforation risk

* Comparing 3 models for geometry:

1) Spherical / rounded tip
2) Flat circular punch
3) Conical tip

DATA & DIAGRAM

- given: insertion force F
- geometry variables: R (sphere), a (flat punch), α & r (cone)
- Youngs modulus: $E \approx \frac{E}{1-\nu^2}$

DATA & DIAGRAM

- given: insertion force F
- geometry variables: R (sphere), a (flat punch), α & r (cone)
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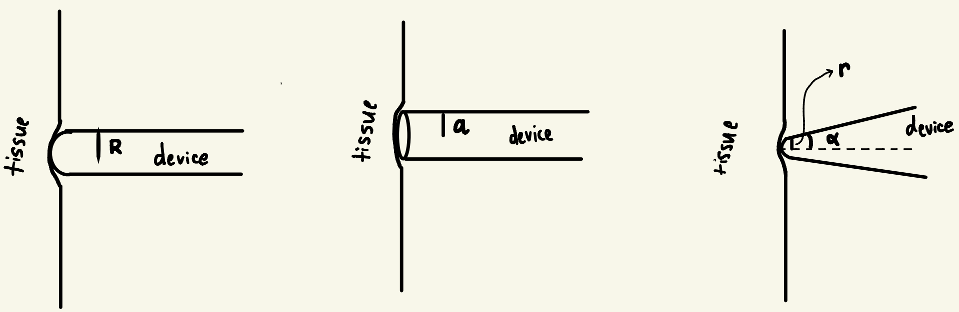
variable	definition
F (N)	insertion force
ν	Poissons ratio of myometrium
E (Pa)	Youngs modulus of myometrium
R (m)	Spherical tip radius
a (m)	flat circular tip radius
α (rad)	angle of conical tip
r (m)	blunt apex radius of conical tip
δ (m)	Indentation depth
$p(r)$ (Pa)	pressure distribution
P_0 (Pa)	peak/center pressure
$\bar{p}$ (Pa)	mean pressure



ASSUMPTIONS

- Elastic half space model: Semi-infinite, homogenous, isotropic, linearly elastic solid
- frictionless normal contact
- Rigid tip
- No adhesion at first touch
- Cone model has a finite apex with radius r

THEORY & EQUATIONS

① Spherical tip

↳ for a rigid sphere on elastic half-space under normal load F:

Hertz load indentation for sphere (Zhu):

$$F = \frac{4}{3} E R^{1/2} \delta^{3/2} , \quad a^2 = R\delta$$

$$F = \frac{4}{3} E R^{1/2} \left(\frac{a}{R}\right)^{3/2} = \frac{4}{3} E \cdot \frac{a^3}{R} \rightarrow a = \left(\frac{3FR}{4E}\right)^{1/3}$$

$$F = \int_0^a 2\pi r p_0 \sqrt{1 - \frac{r^2}{a^2}}\, dr = \frac{2\pi p_0 a^2}{3} \Rightarrow p_0 = \frac{3F}{2\pi a^2}$$

- contact radius: $a = \left(\frac{3FR}{4E}\right)^{1/3}$
- $p(r) = p_0 \sqrt{1 - \frac{r^2}{a^2}}$ for $0 \leq r \leq a$
- $p_0 = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi} \left(\frac{4E}{3FR}\right)^{2/3}$
- $p_0 \propto R^{-2/3}$ ∴ smaller R = higher peak stress

② Flat circular tip

↳ Axis-symmetric flat-ended circular punch (Johnson) (Ling)

- $\bar{p} = \frac{F}{\pi a^2}$, where a = radius

③ Conical tip (Johnson) (Ling)

for a sharp cone:

$$F = \frac{2E}{\pi} \tan\alpha\, \delta^2 , \quad a = \delta \tan\alpha \rightarrow \bar{p} = \frac{F}{\pi a^2} = \frac{\frac{2E}{\pi} \tan\alpha\, \delta^2}{\pi(\delta^2 \tan\alpha)} = \frac{2E}{\pi^2 \tan\alpha}$$

→ real cone tips are blunted, and peak stress increases rapidly as r decreases → $a \sim \sqrt{r\delta}$

SOLUTION

→ Spherical: $p_0 \propto R^{-2/3}$ → Smallest R yields maximum stress

→ Flat circular: $\bar{p} = \frac{F}{\pi a^2}$ → smallest face radius, a, maximizes stress

→ Conical: $a \propto \sqrt{r\delta}$, $\bar{p} = \frac{F}{\pi a^2}$ → Maximizing stress: conical tip with smallest blunt radius

- Sphere vs. flat circular:

$$a = \left(\frac{3FR}{4E}\right)^{1/3} \rightarrow \bar{p} = \frac{F}{\pi a^2} \rightarrow \bar{p}(R) = \frac{F}{\pi}\left(\frac{4E}{3FR}\right)^{2/3} \quad \text{vs.} \quad p_{0,sphere} = \frac{3F}{2\pi}\left(\frac{4E}{3FR}\right)^{2/3}$$

$$p_{0,sphere} = \frac{2}{3}\bar{p}(R)$$

∴ Spherical is 1.5 times the mean pressure of a flat circular tip with the same a

Stress-minimizing model:

- $E_{endometrium}$ = 40 kPa (Manchanda)
- ν = 0.49 (Manchanda)

→ E^* (effective modulus) = $\frac{E}{1-\nu^2}$

- Insertion force = 4 N (Duncan)
- target peak pressure: p_{max} = 100 kPa (Baah-Dwomoh)

$$E^* = \frac{E}{1-\nu^2} = \frac{40000\,Pa}{1-(0.49)^2} = 52,639\,Pa$$

$$p_0 = \frac{3F}{2\pi}\left(\frac{4E^*}{3FR}\right)^{2/3} \rightarrow R = \frac{\left((4E^*)(3F)\right)^{1/2}}{(2\pi p_0)^{3/2}} = \frac{\left((4\cdot 52639\,Pa)(3\cdot 4N)\right)^{1/2}}{2\pi(100000\,Pa)^{3/2}} = 1.464\times10^{-3}\,m = \boxed{1.464\,mm}$$

UNITS:

$$\frac{Pa\cdot\sqrt{N}}{Pa^{3/2}} = \frac{\sqrt{N}}{\sqrt{P}} = \sqrt{\frac{N}{N\cdot m^2}} \stackrel{\checkmark}{=} m$$

* With F = 4N, E = 40 kPa, ν = 0.49, keeping $p_0 < 100$ kPa requires a rounded tip radius of at least R = 1.464 mm

RESULTS / DISCUSSION :

while the geometric analysis showed that the flat cylindrical model would minimize average pressure applied to the myometrium, the spherical model will minimize peak stress. As peak stress is generally more likely to cause the tissue to fail, it will make most sense to use the spherical model and minimize peak stress. For the spherical model, the smallest tip radius maximizes the stress on the tissue. The minimum tip radius necessary to provide less than 100 kPa of pressure was calculated, with an example applied force of 4 N (measured from intrauterine instruments during IUD placement), a youngs modulus of 40 kPa (based on elastography reports on the myometrium), and a maximum peak pressure of 100 kPa (well below reported soft tissue rupture stresses which are often 0.8 - 2.6 MPa in pelvic tissues. This yielded a minimum radius of 1.464 mm, which sets a lower bound for the necessary radius of the instrument.

Citations:

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Engineering Analysis: Determining Dimensions for Fluidic Low-Pass Filter

December 8, 2025

1 Problem

Based on video footage, we were able to determine that we need a cutoff frequency of about 5 Hz for our fluidic low-pass filter. We will use a latex membrane for the capacitor between the input and output channels and a constriction to model a fluidic resistor. We need to determine the length and diameter of the constriction and the diameter of the membrane to achieve the desired cutoff frequency.

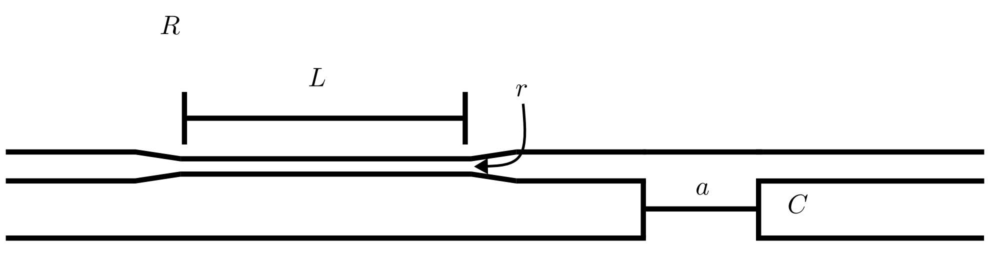

Figure 1: Schematic of fluidic low-pass filter showing relvant dimensions.

2 Data

Here, we assume that:

- $a = 1$ cm for the purposes of fitting the capacitor in our design.
- The latex membrane follows a fourth order elastic model (see Equation 2).

Variable	Description	Value
f_c	Cutoff frequency	5 Hz
a	Radius of membrane	1 cm
T_0	Tension in membrane	605 N/m
μ	Dynamic viscosity of fluid	0.89 mPa·s
r	Radius of constriction	2.2 mm
L	Length of constriction	?

Table 1: Known variables for fluidic low-pass filter design. The known values were taken from measurements from the tubing we are using.

3 Analysis

First, we define the cutoff frequency in terms of the resistance and capacitance:

$$f_c = \frac{1}{2\pi RC} \tag{1}$$

Based on our assumptions, we can get the capacitance using:

$$C = \frac{\pi a^4}{8T_0} \tag{2}$$

The resistance of the constriction can be modeled using Poiseuille's law:

$$R = \frac{8\mu L}{\pi r^4} \tag{3}$$

Combining these equations, we can solve for the length of the constriction:

$$L = \frac{r^4 T_0}{2\pi f_c \mu a^4} \tag{4}$$

Finally, plugging in the known values, we obtain:

$$\boxed{L = 5.1 \text{ cm}} \tag{5}$$

Engineering Analysis - Force Required for Myometrial Perforation

Problem: Determine the maximum force F_c that a hysteroscope sheath can apply to the fundus of the uterus without causing myometrial perforation.

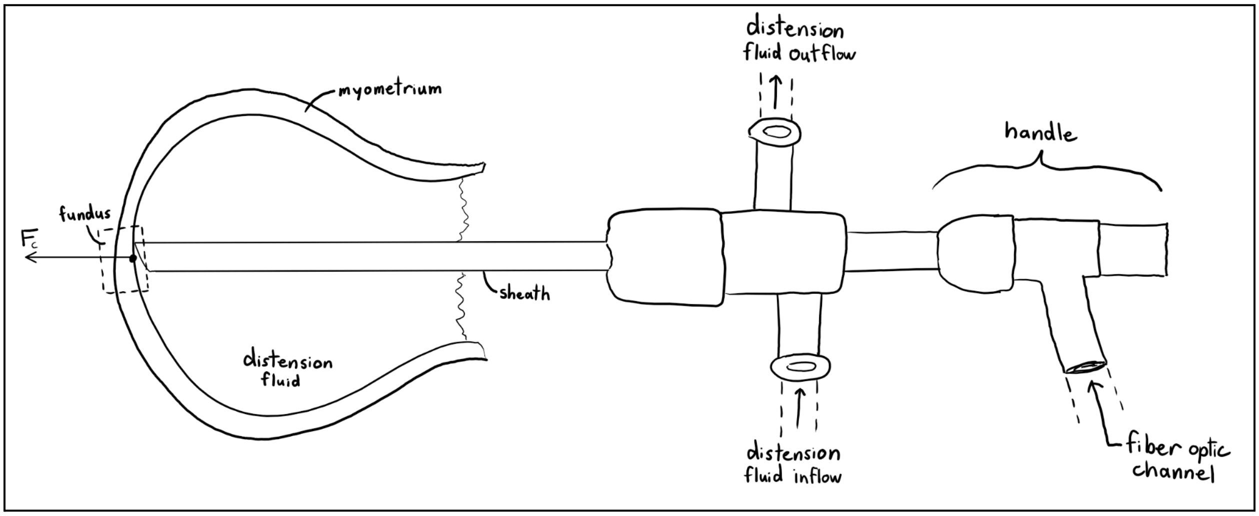

Figure 1. Diagram of a hysteroscope sheath applying a force F_c to the fundus of the uterus.

Data and Diagram:

Variables	Description	Value
E_1	Young's modulus of the sheath	200 GPa → 200×10^9 Pa [1]
E_2	Young's modulus of the uterine wall	?
E_e	Equivalent Young's modulus	?
ν_1	Poisson's ratio of the sheath	0.33 [1]
ν_2	Poisson's ratio of the uterine wall	0.47 [2]
S	Shore A hardness of the uterine wall	10A [3]
P_{max}	Pressure on the uterine wall required for myometrial perforation	2 N/mm^2 → 2×10^6 Pa [4]
$P_{c,\,max}$	Pressure on the uterine wall due to the sheath required for myometrial perforation	?
P_{df}	Pressure on the uterine wall due to distension fluid	100 mmHg → 13.3322×10^3 Pa [5]
D_e	Diameter of the sheath tip	400 μm → 0.0004 m [6]
D_s	Outer diameter of the sheath	9.525 mm → 0.009525 m
L	Length of contact between the sheath and the uterine wall	$\frac{1}{3}\pi D_s = \frac{1}{3}\pi(0.009525\ m) = 0.00997\ m$

F_c	Force on the uterine wall due to the sheath required for myometrial perforation	?

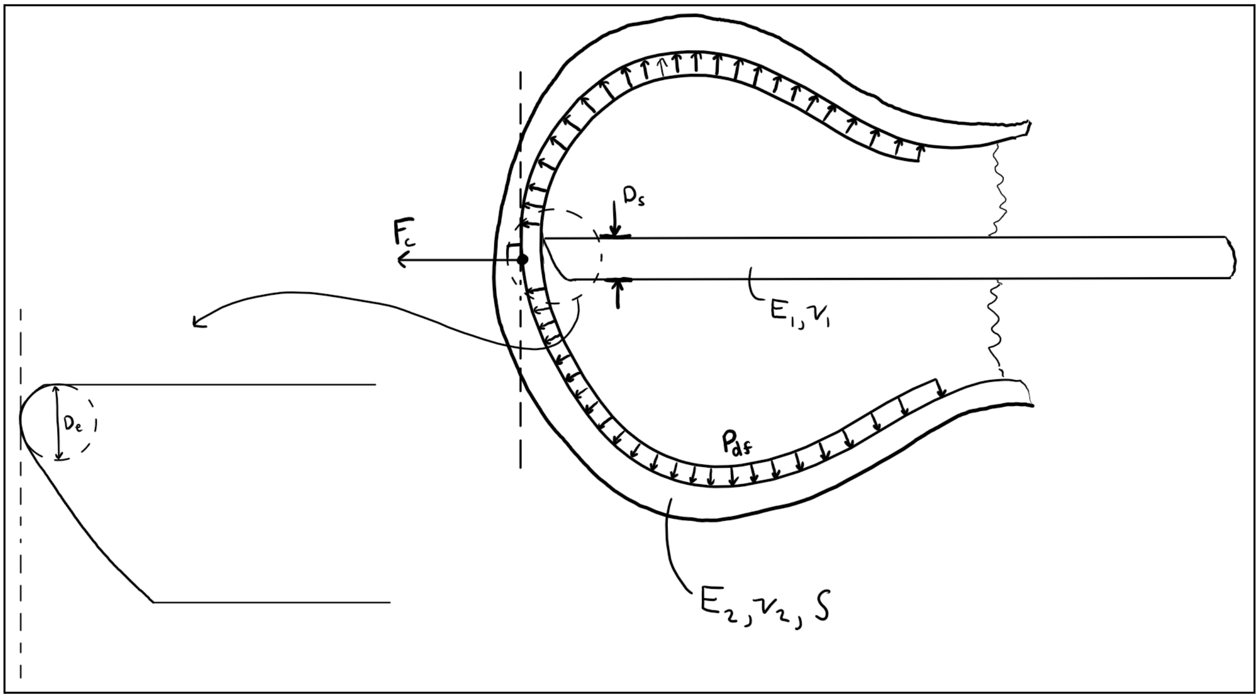

Figure 2. Diagram of a sheath contacting a uterus illustrating the variables of the analysis.

Assumptions:

1. Surfaces in contact are perfectly smooth [1].
2. Materials are homogeneous [1].
3. No frictional forces within the contact area [1].
4. The diameter of the sheath tip is 400 µm [6].
5. The uterine wall can be modeled as soft silicone rubber [7].
6. The sheath is made out of stainless steel.
7. E_1 is significantly larger than E_2.
8. The sheath has an outer diameter of 9.525 mm and a 30° tip.
9. One third of the tip contacts the uterine wall.
10. F_c is normal to the tangent plane of the uterine fundus.

Theory:

- Converting Shore A Hardness to Young's Modulus (in GPa) [8]

$$log(E_2) = 0.0235S - 0.6403 \quad \text{(I)}$$

- Hertzian Contact Stress between a Cylinder and a Flat Surface [1]

$$E_e = \frac{1}{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}} \quad \text{(II)}$$

$$P_{max} = (\frac{2F_c E_e}{\pi L D_e})^{1/2} \quad \text{(III)}$$

- Total Pressure on the Uterine Wall

$$P_{max} = P_{c,max} + P_{df} \quad \text{(IV)}$$

Solution:

1. Isolate for E_2 in equation I and convert from GPa to Pa.

$$E_2 = (10^{0.0235S - 0.6403}) \times 10^6 \quad \text{(V)}$$

2. Reduce equation II, given that $E_1 \gg E_2$.

$$E_e = \frac{1}{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}} \approx \frac{E_2}{1-\nu_2^2} \quad \text{(VI)}$$

3. Substitute equation V into equation VI.

$$E_e \approx \frac{(10^{0.0235S-0.6403}) \times 10^6}{1-\nu_2^2} \quad \text{(VII)}$$

4. Isolate for $P_{c,max}$ in equation IV.

$$P_{c,max} = P_{max} - P_{df} \quad \text{(VIII)}$$

5. Isolate for F_c in equation III.

$$F_c = \frac{P_{c,max}^2 \pi L D_e}{2E_e} \quad \text{(IX)}$$

6. Substitute equations VII and VIII into equation IX.

$$F_c = \frac{(P_{max} - P_{df})^2 \pi L D_e}{2(\frac{(10^{0.0235S-0.6403}) \times 10^6}{1-\nu_2^2})} \quad \text{(X)}$$

7. Solve equation X.

$$F_c = \frac{(2 \times 10^6\ Pa - 13.3322 \times 10^3\ Pa)^2 \pi (0.00997\ m)(0.0004\ m)}{2(\frac{(10^{0.0235(10)-0.6403}) \times 10^6}{1-(0.47)^2})} = 49.002\ N$$

Results and Discussion: Under the assumptions, the sheath would need to apply approximately 49 N to the fundus of the uterus to result in myometrial perforation. This value is roughly 2 times larger than the reported force required for myometrial perforation with a metal sound, a significantly thinner tool [4]. If the outer diameter of the sheath or the diameter of the sheath tip increased, the force required for perforation would increase.

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Engineering Analysis - Spring Constant for Distal Sheath Conformity

Problem: There are two segments of the hysteroscope sheath, a proximal portion and a distal portion. The proximal end of the distal portion of the sheath is constrained within the proximal portion of the sheath. There is a spring within the proximal portion and oriented coaxially with the sheath. The distal end of the spring is attached to the proximal end of the distal sheath. Determine the spring constant k that is needed so that the distal portion of the sheath will be displaced by 50 mm in the proximal direction prior to the sheath achieving the force necessary for myometrial perforation.

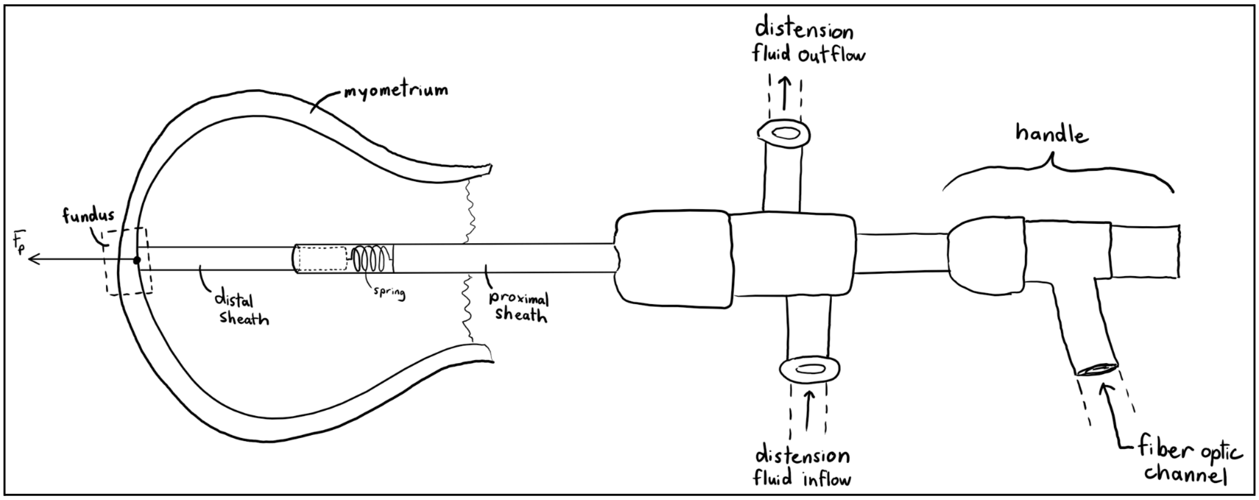

Figure 1. Diagram of a hysteroscope sheath applying a force to the fundus of the uterus. The distal portion of the sheath is connected to the proximal portion via a spring.

Data and Diagram:

Variables	Description	Value
F_p	Force on the uterine wall due to the sheath required for myometrial perforation	49.002 N [1]
F_a	Force applied on the sheath by the user	$F_a \approx 1.5F_p = 73.504\, N$
F_s	Force on the sheath due to the spring	?
l_0	Resting length of the spring	?
l_{min}	Length of the spring during maximum compression	?
Δl	Maximum change in spring length	50 mm → 0.05 m
k	Spring constant	?

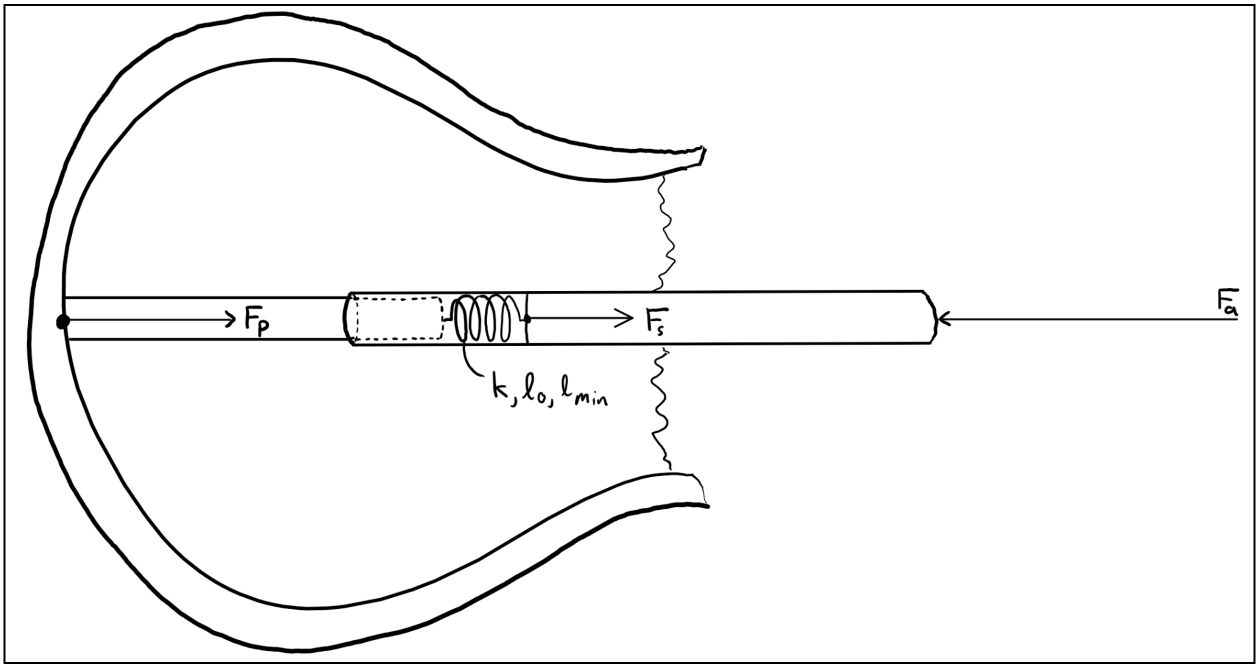

Figure 2. Diagram of a sheath contacting a uterus illustrating the variables of the analysis.

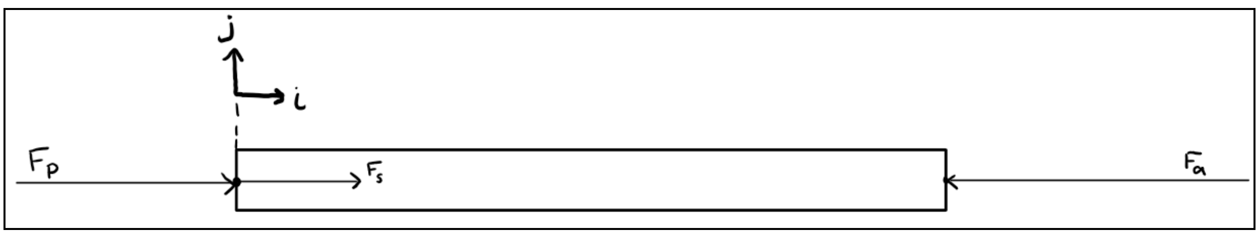

Figure 3. Free body diagram of the proximal segment of the sheath when the spring is fully compressed.

Assumptions:

1. F_p is completely normal to the tangent plant of the uterine fundus.
2. There is no friction between the distal and proximal segments of the sheath.
3. The maximum change in spring length is 50 mm.
4. The spring is massless and obeys Hooke's Law.
5. The sheath is in static equilibrium.

Theory:

- Spring Force at Maximum Compression (I)

  $$F_s = k\Delta l \quad \text{(I)}$$

- Perforation Force (II)

  $$F_p = F_a - F_s \quad \text{(II)}$$

Solution:

1. Isolate for F_s in equation II.
   $$F_s = F_a - F_p \qquad \text{(III)}$$
2. Substitute equation I into equation III.
   $$k\Delta l = F_a - F_p \qquad \text{(IV)}$$
3. Isolate for k in equation IV.
   $$k = \frac{F_a - F_p}{\Delta l} \qquad \text{(V)}$$
4. Solve equation V.
   $$k = \frac{73.504\ N - 49.002\ N}{0.05\ m} = 490.024\ \frac{N}{m}$$

Results and Discussion: Under the assumptions, the spring constant would need to be approximately 490 N/m. A larger spring constant indicates a stiffer spring. If a larger change in spring length was desired, the spring constant should decrease. If a larger applied force is required, the spring constant should increase.

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